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Evolution of the surface magnetostatic wave envelope solitons in a ferromagnet–dielectric–metal structure

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Abstract

Analytical and numerical methods are used to investigate the new localized states of a surface magnetostatic wave envelope in a ferromagnet–dielectric–metal structure in the framework of the extended non-linear Schrödinger equation. The problems of the modulation instability of homogeneous states and long-wavelength modulation of a monochromatic wave in the vicinity of a 'zero-dispersion' point are discussed. The excitation conditions for a multi-soliton state are analysed. It is shown that a certain threshold amplitude of the initial distribution should be reached for the excitation of these states, while no such threshold exists for the one-soliton state. The possibility of the experimental observation of the above-mentioned non-linear states is discussed.

1. Introduction

The investigation of envelope solitons of magnetostatic waves in thin magnetic films has a long history, nearly 20 years (see [1–4], for instance). In this field of research during this period results have been obtained which are valuable not only for the physics of thin films, but for non-linear physics as a whole. In spite of the fact that the investigations have been carried out for a long time, some problems remain unsolved to the present day. One of these problems, which is rather important from our standpoint, is analysed here. As a rule, the model of a single-layer magnetic film is used for the interpretation of experimental results. However, for unavoidable technological reasons the real object of investigations in the experiments is a three-layered ferromagnet–dielectric–metal structure. It possesses a unique non-monotone spectrum of spin waves which differs substantially from that of a single-layer film. It is reasonable that the theoretical model of weakly non-linear dynamics used for adequate interpretation of the experimental data must take account of the fine structure of the linear spin excitation spectrum. Moreover, the spatial dispersion in this structure depends essentially on the thickness of the dielectric layer (the so-called 'size effect') and it can be controlled by external magnetic field. This circumstance allows us to treat a ferromagnet–dielectric–metal structure as an ideal model

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system for the investigations of non-linear (in particular, soliton-like) regimes of magnetostatic wave propagation in systems with variable dispersion parameters.

The papers concerning ferromagnetic dynamics of layered systems known at the present time can be grouped into two categories. In one of them, the dispersion features are analysed, and also peculiarities of the propagation and transformation of spin waves in a linear regime are studied, both taking account of electromagnetic retardation and without it (see, for instance, [5–10]). The other group deals with weakly non-linear dynamics and studies the conditions of the origination, interaction and evolution of the one- and multi-soliton states of the magnetostatic wave envelope [11–14]. For example, in [11, 12] in the framework of the extended non-linear Schrödinger equation (ENSE), taking into consideration both the third-order dispersion and dispersion of the non-linear term, the conditions for the formation, interaction and evolution of volume magnetostatic wave envelope solitons are studied in the layered isotropic ferromagnet–dielectric–metal structure magnetized both normally [11] and tangentially [12].

It was shown [11] that for the definite wavelength of the carrying forward volume magnetostatic wave and for the definite ferromagnet-to-dielectric thickness relation the necessary conditions for the origination of 'dark' and 'bright' Potasek–Tabor solitons may be realized. For the tangentially magnetized plate, it was found [12] that the transformation of the spectrum of the backward volume spin waves takes place not only at the variations of the dielectric layer thickness, but also at the variations of the angle between the direction of wave propagation and the dc magnetic field. These spectrum peculiarities allow us to control the non-linear scenario of wave propagation more effectively, with the help of the above-mentioned parameters. The 'bright soliton' regime may be changed to the 'dark soliton' and vice versa. It was noted also that in the range of large wavelengths in the tangentially magnetized structure, contrary to the normally magnetized structure, the 'dark' but not 'bright' Potasek–Tabor solitons are realized. There are few experimental works on the non-linear dynamics of ferromagnet–dielectric–metal structures among the investigations in the field of thin magnetic films, and here the paper [13] attracts attention. The results of experimental investigations into the non-linear properties of the pulses of the surface magnetostatic wave in this structure are discussed there. In particular, it was revealed that a magnetostatic surface spin wave with the wavelength comparable with the thickness of the dielectric layer Δ is unstable with respect to both automodulation and parametric excitation of spin waves. We call attention to the fact that the value $\lambda \approx \Delta$ in the range of the magnetic field that is mentioned in [13] corresponds to the 'zero-dispersion' point ($\partial^2\omega/\partial k^2 \approx 0$, where ω is the frequency and k is the wavenumber). The spatial dispersion of the group velocity of the magnetostatic waves is absent in this point, and hence the dispersion spreading out of the pulse transmitting along the system is lower in this regime. The 'zero dispersion' is one of the key problems in non-linear optics now. Moreover, the wavelength at which this point appears is the performance characteristic of the waveguide fibre [15]. The detailed comparative analysis of the non-linear and dispersion properties of waveguide fibres and thin magnetic layers was carried out in [16]. Usually, the fibre waveguides are treated as weakly non-linear media. In contrast to this, the magnetic films are systems with higher-order dispersion and significantly non-linearity. The presence of the 'zero-dispersion' point in the range of real values of the physical parameters is a remarkable occurrence. It allows us to reduce a number of mechanisms responsible for the dispersion spreading out from the travelling magnetostatic pulses in the definite range of the wavelength in the ferromagnet–dielectric–metal structure.

In the present paper the conditions for the origination, existence, interaction and evolution of one- and multi-soliton states of the envelope of surface magnetostatic waves are studied, taking account of the fine structure of the spectrum in the ferromagnet–dielectric–metal structure. To study this we use the ENSE model and involve numerical methods. It should be noted that one of the first attempts to solve the Cauchy problem for this type of equation was undertaken in [17]. The equation was solved numerically and all coefficients of the equation were put equal to unity. It was shown that, if the amplitude of the initial pulse exceeds some threshold values, several soliton-like states are excited in the system, but the question of the properties of this state remain open.

The paper consists of the introduction, five further sections and the conclusion. In section 2 the geometry, ground state and peculiarities of the magnetostatic surface wave spectrum are discussed for this structure magnetized in the surface plane. Here also the conditions are formulated for the 'zero-dispersion' point realization, provided that $\lambda \approx \Delta$, and also the values of the external dc magnetic field, and the values of the ratio Δ/d (where d is the thickness of magnetic layer) are presented. In section 3 the evolution equation for the envelope of surface magnetostatic waves is obtained and the problem of modulation instability of a uniform state is discussed. The stability of a plane wave with the wavevector being close to its value in the 'zero-dispersion' point is considered in section 4. The long-wavelength modulations of a monochromatic wave in the vicinity of this point are also investigated. It is shown that the low-amplitude modulation of spatially non-localized non-linear waves with the wavelength $\lambda < \Delta$ may lead to the formation of specific 'dark' quasi-solitons (the weakly damped soliton in the background of the wave [18, 19]). Their evolution is described by the Korteveg–de Vries (KdV) equation. In section 5 some exact solutions of ENSE describing isolated waves with the profile remaining constant during propagation, i.e. 'dark' and 'bright' solitons, are considered. The results of numerical analysis of the conditions for formation of multi-soliton states are also presented. It is shown that there exists an initial distribution amplitude threshold for the excitation of multi-soliton states, which is absent for one-soliton ones.

2. Ground state and spectrum of magnetostatic surface waves

Let us consider the part of the non-linear processes realized in this system that are connected with the dependence of the dispersion relation for magnetostatic waves on the amplitude, as was done in [11, 12]. The input equations for the spectrum calculation are the Landau–Lifshitz equation and magnetostatic equations with corresponding boundary conditions (see, for instance, [20]). The resulting boundary value problem is solved for three regions of a film, namely the magnetic, dielectric and metal layers. It is known that the magnetostatic approximation (the neglected electromagnetic retardation) allows us to simplify the dispersion relation and to write them down in a form convenient for the analysis of weakly non-linear dynamic effects [21]. As a rule, this approach is used in the case of small dielectric permeability values for a ferromagnetic film and surrounding media in the range of not so large frequency values [5]. In the case under consideration the domain of existence for the waves studied lies lower than 5 GHz (see below). We hope that the magnetostatic approximation will have enough reliability for qualitative analysis of weakly non-linear processes in the studied system.

As indicated above, we shall consider a three-layered plate with the ferromagnet–dielectric–metal structure (see figure 1). In practice, the structure studied in the present paper is composed of yttrium iron garnet (YIG) film, separated from a conducting base plane by a dielectric layer. We suppose the magnetic layer to be isotropic and all plates, including the magnetic layer, to be infinite in the plane (z, y). The isotropic model of the ferromagnet restricts the possibility of applying the obtained results for the analysis of experimental data on the evolution of the soliton envelope for the surface magnetostatic waves. For a more adequate description of the features of the propagation of non-linear excitations it is necessary to take account of the effects of magnetocrystalline anisotropy for the ferromagnetic layer. In our opinion, the model of an isotropic magnet is sufficient for the qualitative analysis of the

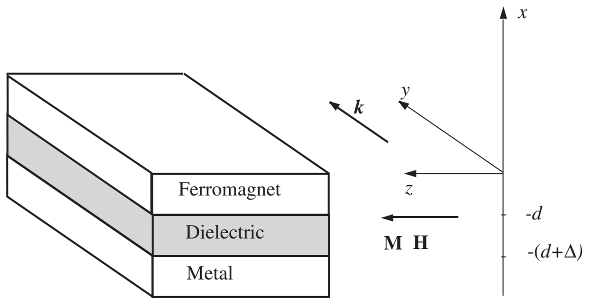

Figure 1. The coordinate system.

phenomena considered (see [7–9]). Let the thickness of the ferromagnetic layer be d and let this layer be separated from the metallic one by a dielectric interlayer with the thickness Δ. For the sake of simplicity, the dielectric is considered to have the same dielectric constant as the magnetic film. The metallic plane is assumed to be ideally conducting and the normal component of magnetic induction vanishes on the conducting surface. We assume also that the wave propagates parallel to the conducting plane along the y axis and that the dc magnetic field is perpendicular to the direction of wave propagation. In this statement, the problem of obtaining the dispersion relation for magnetostatic surface waves in a three-layer isotropic ferromagnet–dielectric–metal film was solved in [9]. The extension of these results to the magnetic layer of finite thickness (bounded in the z direction) presents no special problems and was done in [10].

The dispersion relation for surface magnetostatic waves obtained in [9] for the case under consideration has the form

$$\Omega s = -[C(2-B)-A] \pm \{[C(2-B)-A]^2 + 4(2BC)[C(2B\Omega_H^2 + \Omega_H(2+B)+1) - (1+A\Omega_H)]\}^{1/2}[2(-2BC)]^{-1}, \quad (1)$$

where

$$A = 1+\tanh(-|k|\Delta), \qquad B = 1-\tanh(-|k|\Delta), \qquad C = \exp(2|k|d),$$
$$\Omega = \frac{\omega}{\omega_M}, \qquad \Omega_H = \frac{\omega_H}{\omega_M}, \qquad \omega_H = \gamma H, \qquad \omega_M = 4\pi\gamma M_0, \quad (2)$$

where ω is the circular frequency, γ is the gyromagnetic ratio, H is the dc magnetic field and M_0 is the saturation magnetization, $s = k/|k|$. Here the frequency of the magnetostatic surface waves lies within the range

$$[\Omega_H(\Omega_H+1)]^{1/2} \leqslant \Omega \leqslant \Omega_H + \tfrac{1}{2}. \quad (3)$$

Let us evaluate this range for the plate with the following material parameters: $d = 10\ \mu$m, $\Delta = 25\ \mu$m, $H = 200$ Oe, $4\pi M_0 = 1750$ G. The upper frequency limit for this range is equal to 3.2 GHz and the lower frequency limit is 1.7 GHz. It is clear that the limiting frequency values increase with the dc magnetic field.

Let us note some limiting cases: $\Delta \to \infty$ (metallic layer is absent) and $\Delta \to 0$ (dielectric layer is absent). In the first case the dispersion relation (1) can be recast in the form

$$\exp(-2|k|d) = 1 + 4\Omega_H + 4\Omega_H^2 - 4(\Omega s)^2, \qquad s^2 = 1 \quad (4)$$

which is exactly the same as that obtained by Daemon and Eshbah [22]. The surface wave is degenerate because the dispersion relations for the $s = +1$ and -1 cases coincide. In the absence of a dielectric layer the relation (1) has the following form:

$$\exp(2|k|d) = \frac{1+\Omega s+\Omega_H}{-2\Omega^2 + \Omega s + (\Omega_H+1)(2\Omega_H+1)}. \quad (5)$$

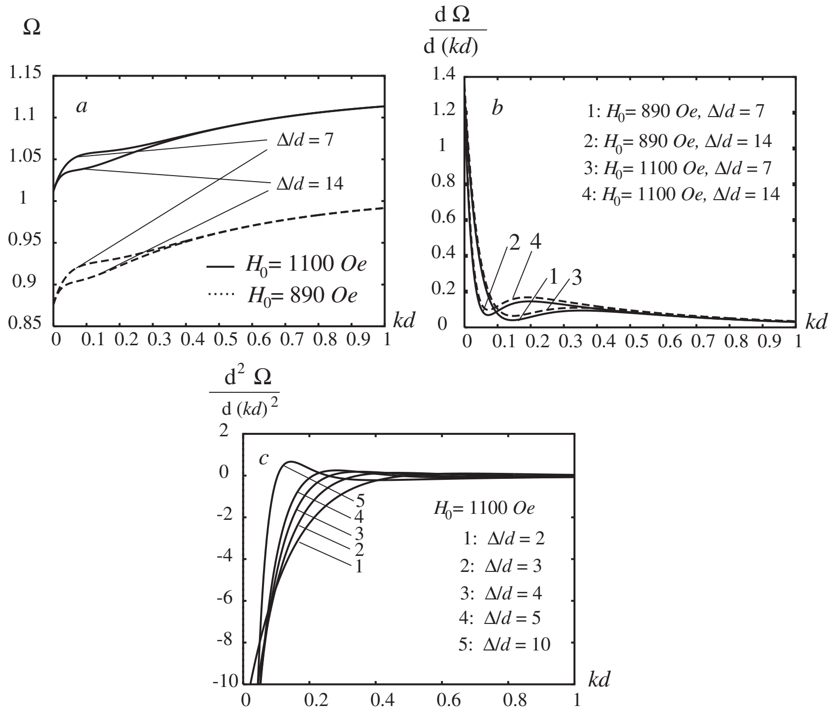

Figure 2. The spectrum of surface magnetostatic waves (a), the first derivative of the dispersion relation with respect to the wavenumber (b) and the second derivative (c) for several values of the parameter Δ/d and the dc magnetic field: $M_0 = 1740$ G, $d = 14$ μm.

Apparently, the expression of this type for a surface wave spectrum was obtained first by Seshadry [23] in the analysis of dipole resonance peculiarities for plane wave scattering in axially magnetized ferrite poles with an ideally conducting kernel. The fact that the parameter s appears in the first power means that the waves propagating in the $+y$ and $-y$ directions have different characteristics (i.e. velocities), so that the degeneracy of the surface waves taking place in the case $\Delta \to \infty$ is eliminated here.

Let us consider the forward (that means, having the positive projection of group velocity v_g on the direction of phase velocity v_{ph}) surface wave propagating in the $+y$ direction ($s = +1$). Exactly for this wave the 'dispersionless' regime of linear wave propagation was experimentally observed in the range between 2350 and 2550 MHz [9]. Figure 2(a) presents the dispersion relation $\Omega\ (kd)$ for various values of the dc magnetic field H, the dielectric-to-ferromagnet thickness ratio Δ/d and the following values of material parameters: $d = 14$ μm, $4\pi M_0 = 1750$ G. The first derivative of the dispersion $\Omega\ (kd)$ with respect to (kd) is given in figure 2(b) and the second derivative is given in figure 2(c) for the same H values and Δ/d ratio. It follows from figure 2(c) that the dispersion curve has 'zero-dispersion' points in which $\frac{\partial^2\Omega}{\partial(kd)^2}(k_0 d) = 0$. For some curves there is one such point, while for others there are two of them, with the second point lying in the range where (kd) is comparable to unity. As was noted in the introduction, the left 'zero-dispersion' point may appear at $\lambda \approx \Delta$. Figure 3(a) displays the surface $\frac{\partial^2\Omega}{\partial(kd)^2}|_{k=\Delta^{-1}}$ in the space of the variables Δ/d and the dc magnetic field H. The bold broken curve in this figure arises from the intersection of this surface with the plane $\frac{\partial^2\Omega}{\partial(kd)^2}|_{k=\Delta^{-1}} = 0$. This curve is shown in figure 3(b) individually. Such a simple dependence of

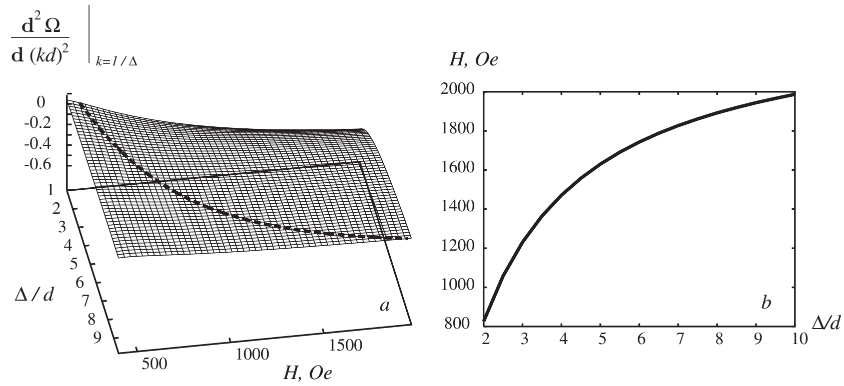

Figure 3. The dependence of group velocity dispersion on the value of magnetic field H_0 and on the ratio Δ/d at fixed wavenumber $k = \Delta^{-1}$, for the broken curve $\frac{\partial^2\Omega}{\partial k^2}|_{k=\Delta^{-1}} = 0$ (a); the curve $H_0(\Delta/d)$ whose points satisfy the condition $\frac{\partial^2\Omega}{\partial k^2}|_{k=\Delta^{-1}} = 0$ (b) and which corresponds to the curve in (a).

the wavevector corresponding to the 'zero-dispersion' point on the thickness of the interlayer $k \approx \Delta^{-1}$ takes place only for the surface magnetostatic waves in a ferromagnet–dielectric–metal structure for the geometry depicted in figure 1. For example, if the structure is taken with a dielectric-to-ferromagnet thickness ratio $\Delta/d = 2$ and the dc field $H_0 = 850$ Oe, the 'zero-dispersion' point has the following parameters: $kd = (\Delta/d)^{-1} = 0.5$, $\omega = 4.81$ GHz. In the case of volume magnetostatic waves both in normally and tangentially magnetized structures (see [11, 12]) this dependence is more complicated. The dependence $H(\Delta/d)$ given in figure 3(b) makes it possible to set external parameters H and Δ/d in such a way that the 'zero-dispersion' point is realized for the wavelength $\lambda = \Delta$. Note that the dependence $H(\Delta/d)$ practically does not change if the magnetic layer has finite width. To prove this statement we have used the spectrum of surface magnetostatic waves obtained in [10], which we do not present here. The evolution of an envelope soliton in the 'zero-dispersion' point, as is well known, is described by ENSE with third-order dispersion (see, for example, [18]) and has some interesting features.

3. Equation of evolution and the modulation instability

The basic non-linear effect analysed in the present paper is the dependence of the magnetostatic surface wave frequency Ω on the oscillation amplitude φ. The non-linearity enters into the dispersion relation (1) through the z component of the magnetization. In the local frame of (xyz) its equilibrium direction coincides with the z axis. For small deviations from the equilibrium value $M = M_0$ (where M_0 is the saturation magnetization) it is defined by

$$M_z = M_0\{1 - (M_x^2 + M_y^2)/(2M_0^2)\}^{1/2} = M_0(1 - |\varphi|^2). \tag{6}$$

As a consequence, the dispersion law (1) can be represented in the form of the relation

$$G(k, \Omega, |\varphi|^2) = 0 \tag{7}$$

which permits us to write down the non-linear equation that defines the function $\varphi(r, t)$ as an envelope of the carrier wave, using the procedure proposed in [21, 24]. As a result we obtain the following equation for this function:

$$\mathrm{i}\left(\frac{\partial\varphi}{\partial t} + v_{\mathrm{g}}\frac{\partial\varphi}{\partial x}\right) + \frac{1}{2}D_\omega\frac{\partial^2\varphi}{\partial x^2} - \mathrm{i}\alpha_\omega\frac{\partial^3\varphi}{\partial x^3} - N_\omega|\varphi|^2\varphi + \mathrm{i}Q_\omega\frac{\partial}{\partial x}(|\varphi|^2\varphi) = 0 \tag{8}$$

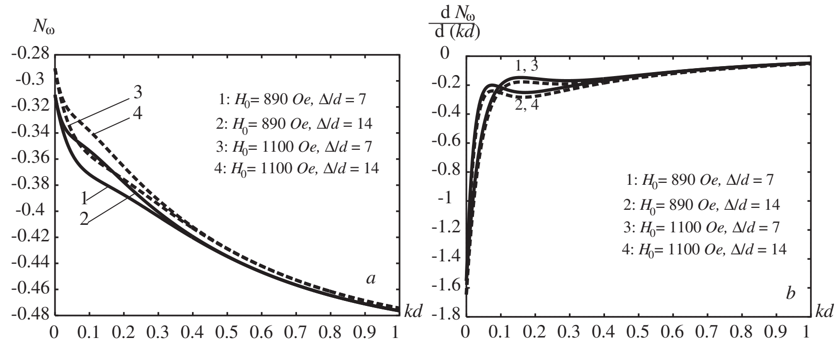

Figure 4. The wavenumber dependence of the non-linearity coefficient N_ω (*a*), and that of the non-linearity dispersion Q_ω (*b*); the parameters of the curves correspond to figure 2(*a*).

where $v_g = \partial\Omega/\partial k(k, \varphi = 0)$ is the group velocity, $D_\omega = \partial^2\Omega/\partial k^2$, $\alpha_\omega = \frac{1}{6}\partial^3\Omega/\partial k^3$, $N_\omega = \partial\Omega/\partial|\varphi|^2$ and $Q_\omega = \partial N_\omega/\partial k$. Deducing equation (8) we have assumed that the dispersion relation (7) is resolved with respect to the frequency Ω as a function of k and $|\varphi|^2$. The first term (in the parentheses) describes the waves that propagate with a group velocity v_g in the linear medium without dispersion. The second (second-order dispersion) and the third (third-order dispersion) are responsible for the dispersion broadening of the packet, the coefficient N_ω is responsible for non-linearity and Q_ω describes the dispersion of the non-linear term.

Note that the term with the coefficient α_ω is especially important in the vicinity of the 'zero-dispersion' point. The coefficient N_ω is negative in the considered problem for $kd \leqslant 1$ (see figure 4(a)). The non-linearity dispersion term that is proportional to $Q_\omega = \partial N_\omega/\partial k$ is also negative in this range (see figure 4(b)). Note also that, according to figure 2, the quantities D_ω and α_ω can be positive or negative. The first, second and fourth terms correspond to the classical entirely integrable non-linear Schrödinger equation (NSE). Besides Ω, the following quantities in equation (8) are dimensionless: $x \to x/d, t \to t\omega_M, k \to kd$. After passing over to the frame of reference moving with the group velocity of linear waves with $x \to X - v_g t$, and after a linear transformation $\varphi(x, t) = A\Phi(mX, t)$, where $m = |D_\omega|^{-1/2}$, $A = |N_\omega|^{-1/2}$, $\alpha_3 = -\alpha_\omega|D_\omega|^{-3/2}$ and $\alpha_1 = Q_\omega|D_\omega|^{-1/2}|N_\omega|^{-1}$ equation (8) can be transformed into

$$\mathrm{i}\frac{\partial\Phi}{\partial t} + \mathrm{sgn}(D_\omega)\frac{1}{2}\frac{\partial^2\Phi}{\partial X^2} + \mathrm{i}\alpha_3\frac{\partial^3\Phi}{\partial X^3} + \mathrm{i}\alpha_1\frac{\partial}{\partial X}(|\Phi|^2\Phi) + |\Phi|^2\Phi = 0 \qquad (9)$$

where the sign in the second term is determined by the sign of the second derivative $\partial^2\Omega/\partial k^2$. As a rule, the equation of the type (9) is called the ENSE [25, 26]. For arbitrary relations between its coefficients this equation is not entirely integrable. Note that at the present time some particular cases are known in which this equation is entirely integrable (see, e.g., [27, 28]).

Unfortunately, the procedure for obtaining the ENSE used above gives a rather rough model of non-linear and dispersion effects. It has a limited area of applicability. At first, it may be used only for small deviations of magnetization from its equilibrium position ($|\varphi| \ll 1$). Secondly, it cannot be used for $kd \to 0$, because for such kd the spectrum of linear spin excitations of the structure considered is not an analytical function of kd ($\omega(k) \sim |kd|$). And, finally, in this approach the non-local character of the magnetic dipole–dipole interaction is not accounted for. It is known that taking account of non-locality leads to a non-linear integral-differential evolution equation. The problem of damping is extremely important here. In this context it is necessary to consider both uniform and non-uniform damping in the modelling of magnetic film dynamics. Moreover, non-uniform terms prevail sometimes [29].

In spite of these circumstances, in most of the experimental work on solitons in thin magnetic films known today the interpretation of results is based essentially on the approach proposed in [21, 24]. Here the remarkable papers of Bordman with co-workers [30, 31], Patton [2] and Kalinikos [1] can be mentioned. At the end of the 1990s, we suggested an alternative approach to the derivation of evolution equations in thin magnetic films (not layered) [32], which allows us to take account correctly of the analytical peculiarities of the linear spectrum of spin excitations and long-range magnetostatics interaction character. Nevertheless, in spite of its limitations the heuristic approach used to derive the evolution equation for the envelope of the carrying wave (8) has its validity for finite values of kd and rather small amplitudes φ. Note also that equation (8) has been used recently for the interpretation of experimental data on the evolution of solitons in single-layer thin magnetic films of iron garnets [33].

We shall analyse below the conditions of instability of the envelope wave and also how the initial distribution evolves with time. Let us consider at first the simplest case of the homogeneous initial state. For this case, we linearize equation (9) in the vicinity of some solution independent of the coordinate X. This solution can be easily found:

$$\Phi_0(t) = A_0 \exp(\mathrm{i}|A_0|^2 t), \tag{10}$$

where A is an arbitrary complex number. Assume

$$\Phi(X, t) = \Phi_0(t)(1 + \varepsilon(X, t)) \tag{11}$$

and linearize the ENSE with respect to $\varepsilon(X, t)$. It is easy to show that the equation for $\varepsilon(X, t)$ in this case has the form

$$\mathrm{i}\frac{\partial \varepsilon}{\partial t} + \mathrm{sgn}(D_\omega)\frac{1}{2}\frac{\partial^2 \varepsilon}{\partial X^2} + \mathrm{i}\alpha_3\frac{\partial^3 \varepsilon}{\partial X^3} + |A_0|^2(\varepsilon + \varepsilon^*) + \mathrm{i}\alpha_1|A_0|^2\left(2\frac{\partial \varepsilon}{\partial X} + \frac{\partial \varepsilon^*}{\partial X}\right) = 0. \tag{12}$$

We shall look for the solution of (12) in the form $\varepsilon = b_1 \exp[\mathrm{i}(\tilde{\Omega}t + KX)] + b_2 \exp[-\mathrm{i}(\tilde{\Omega}^* t + KX)]$, which leads to the dispersion relation

$$\tilde{\Omega} = -\alpha_3 K^3 + 2\alpha_1 |A_0|^2 K \pm \{K^2[\tfrac{1}{4}K^2 - |A_0|^2(\sigma - |A_0|^2\alpha_1^2)]\}^{1/2}, \tag{13}$$

where $\sigma = \pm 1$, depending on the sign '+' or '−' of the second derivative $\frac{\partial^2 \Omega}{\partial k^2}$. The modulation instability ($\mathrm{Im}\,\tilde{\Omega} \neq 0$) appears for the perturbations with wavevectors varying in the range

$$0 < K^2 < 4|A_0|^2(\sigma - |A_0|^2\alpha_1^2). \tag{14}$$

For $K = K_0$, that is determined by the relation

$$K^2 = 2|A_0|^2(\sigma - |A_0|^2\alpha_1^2), \tag{15}$$

one may expect the maximum rate of developing modulation instability. It follows from the relations (14) and (15) that the considered modulation instability is developed only for $\sigma = 1$, provided the following inequality $|A_0|^2\alpha_1^2 < 1$ (or $|A_0|^2 Q_\omega < N_\omega^2 D_\omega$) is satisfied. For instance, for the point of the spectrum $kd = 0.195$ with the proviso that the field value $H_0 = 1100$ Oe, $\Delta/d = 100/14$ and the coefficients α_1 and α_3 are equal, respectively, to -0.59 and -0.46, this inequality is easily satisfied, because $|A_0|^2 \approx |\varphi|^2$, and $|\varphi|^2 \ll 1$. The initial stage of the development of the modulation instability from the homogeneous state is described by ENSE for the mentioned values of spectral parameters and is presented in figure 5. The initial distribution in this case was chosen in the form

$$\Phi(X) = A_0(1 + \delta \cos K_0 X), \tag{16}$$

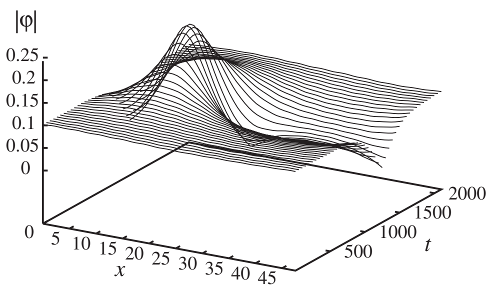

Figure 5. The initial stage of the modulation instability development.

where $A_0 = 0.2$ and $\delta = 0.01$. The interval of calculation was selected to be equal to the wavelength $\lambda_0 = 2\pi/K_0$ of the most unstable harmonic (for the parameters taken it was $\lambda_0 \approx 3.14 \times 10^{-2}$ cm). The initial stage of the development of modulation instability explicitly demonstrates the appearance of a localized soliton state. Unfortunately, we cannot determine its parameters with high enough accuracy to relate these states to currently known ENSE solitons, for example [25, 27, 28].

4. Long-wavelength modulations of monochromatic non-linear waves in the vicinity of the zero-dispersion point

To make clear the peculiarities of the weakly non-linear dynamics of magnetostatic surface waves near the 'zero-dispersion' point let us consider the stability condition for the plane wave:

$$\Phi_0(X, t) = w_0^{1/2} \exp(\mathrm{i}(\Omega_0 t + k_0 X)) \tag{17}$$

which is an exact solution of the equation

$$\mathrm{i}\frac{\partial \Phi}{\partial t} + \mathrm{i}\frac{\partial^3 \Phi}{\partial X^3} + \mathrm{i}\alpha_2 \frac{\partial}{\partial X}(|\Phi|^2 \Phi) + |\Phi|^2 \Phi = 0 \tag{18}$$

where $\Phi(nx, t) = A\varphi(x, t)$, $A = N_\omega^{-1/2}$, $n = -\alpha_\omega^{-1/3}$ and $\alpha_2 = Q_\omega n A^2$. Equation (8) can be reduced to model (18) in the vicinity of the 'zero-dispersion' point. Here in (17) w_0 is the amplitude and k_0 is the wavenumber corresponding to the deviation of the wavevector from its value at the point where $D_\omega = 0$. The values of α_ω, N_ω and Q_ω are taken at the point of zero dispersion. The dispersion relation for the wave (17) has the form

$$\Omega_0 = k_0^3 + w_0(1 - \alpha_2 k_0). \tag{19}$$

For analysis of the stability conditions let us assume $\Phi = \Phi_0(1 + \tilde{\Phi})$, where $\tilde{\Phi} \ll 1$. It is determined by the equation

$$\mathrm{i}\frac{\partial \tilde{\Phi}}{\partial t} - 3\mathrm{i}k_0^2 \frac{\partial \tilde{\Phi}}{\partial X} - 3k_0 \frac{\partial^2 \tilde{\Phi}}{\partial X^2} + \mathrm{i}\frac{\partial^3 \tilde{\Phi}}{\partial X^3} + w_0(1 - \alpha_2 k_0)[\tilde{\Phi} + \tilde{\Phi}^*] + \mathrm{i}\alpha_2 w_0 \left[2\frac{\partial \tilde{\Phi}}{\partial X} + \frac{\partial \tilde{\Phi}^*}{\partial X}\right] = 0. \tag{20}$$

The linear analysis shows that the plane wave (17) is modulation unstable (the exponential increase of the amplitude) for $k_0 < 0$, while for $k_0 > 0$ the ground state is stable. In the latter case the Goldstone modes may exist against the background of the plane wave with the dispersion law $\lambda_i(p)$, $i = 1, 2$:

$$\lambda_i(p) = (3k_0^2 - 2\alpha_2 w_0 + \varepsilon_i m)p + (1 + \varepsilon_i n)p^3 \tag{21}$$

where $\varepsilon_1 = 1$, $\varepsilon_2 = -1$, $m = \{6k_0 w_0(1 - \alpha_2 k_0) + \alpha_2^2 w_0^2\}^{1/2}$ and $n = 9k_0^2/(2m)$. We have assumed in deriving (21) that $(k_0 p)^2 \ll m^2$. Taking into account that the wavevector for magnetostatic surface waves at the point of zero dispersion is equal to Δ^{-1} (see section 2) we can conclude that the modulation instability of the system takes place if the wavelength satisfies $\lambda > \Delta$. On the other hand, the plane wave (17) may be stable if $\lambda < \Delta$, so that Goldstone's modes may exist against its background. However, we must keep in mind that the wavenumber k_0 should be taken in such a way that the inequality $|\partial^2\Omega/\partial k^2| \ll |\partial^3\Omega/\partial k^3|$ will not be violated.

Let us consider long-wavelength low-amplitude modulations of the wave (17). The corresponding solution of equation (18) is presented in the form

$$\Phi(X, t) = w^{1/2} \exp(\mathrm{i}(\Omega_0 t + k_0 X + \chi)) \tag{22}$$

where $w = w_0(1 + V(X, t))$, $|V(X, t)| \ll 1$, $k_0 \gg |\frac{\partial\chi}{\partial X}|$ and $\Omega_0 \gg |\frac{\partial\chi}{\partial t}|$. We shall write down the set of coupled equations for the fields V and χ within the quadratic terms in V and χ. In the long-wavelength limit in terms linear in χ and V in those equations the derivatives with respect to spatial coordinates of orders higher than fourth may be neglected. Only low-order derivatives will be retained in the non-linear terms. As result, we obtain the following set of equations:

$$\begin{aligned} &\hat{L}V - 6k_0\chi_{XX} - 3\partial_X[(\chi_X)^2 + 2k_0 V\chi_X] + \alpha_2 w_0 \partial_X[2V + \tfrac{3}{2}V^2] = 0, \\ &\hat{L}\chi + \hat{M}V - 3k_0(\chi_X)^2 + \alpha_2 w_0 \chi_X V = 0 \end{aligned} \tag{23}$$

where

$$\hat{L} = \hat{L}_0 + \partial_X^3, \qquad \hat{L}_0 = \partial_t - (3k_0^2 - \alpha_2 w_0)\partial_X, \qquad \hat{M} = -1 + \tfrac{3}{2}k_0\partial_X^2. \tag{24}$$

The field V can be easily eliminated from (23). The long-wavelength-limit relation between fields V and χ in the principal approximation is used to transform the non-linear terms:

$$V(1 - \alpha_2 w_0) = \hat{L}_0\chi. \tag{25}$$

After simple calculations a closed equation for $\chi(X, t)$ is obtained:

$$\begin{aligned} &\{-\hat{L}_0^2 - 2\hat{L}_0\partial_X^3 + 6k_0\partial_X^2 - 9k_0^2\partial_X^4 - 2\nu\partial_X\hat{L}_0 + 3\nu\partial_X^3\hat{L}_0\}\chi \\ &\qquad + 3\partial_X\{\chi_X^2 + 2(\alpha_2 w_0)^{-1}\nu k_0 \chi_X(\hat{L}_0\chi)\} - \nu\{(\hat{L}_0\chi_X)(\hat{L}_0\chi) + \chi_X(\hat{L}_0^2\chi)\} \\ &\qquad - \tfrac{3}{2}\nu^2(\alpha_2 w_0)^{-1}(\hat{L}_0\chi)^2 + 3k_0\hat{L}_0\chi_X^2 = 0 \end{aligned} \tag{26}$$

where $\nu = (\alpha_2 w_0)/(1 - \alpha_2 w_0 k_0)$. In deducing (26) we neglect the terms that are cubic in the fields V and χ in the dynamic equation. The analysis shows that this is appropriate for waves with the stationary profile travelling with the velocity u, if the following inequality is satisfied:

$$(6 + 6k_0 l[2\nu(\alpha_2 w_0)^{-1} + 1] - 3\nu l^2[\nu(\alpha_2 w_0)^{-1} + 1])(1 - \alpha_2 w_0 k_0) \gg |lA| \tag{27}$$

where A is a typical amplitude of the field χ_X, $l = u - 3k_0^2 + \alpha_2 w_0$.

Let us consider the solutions of equation (26). Assuming

$$\chi = \chi(\xi), \qquad \partial_\xi\chi = f(\xi), \qquad \xi = x + ut \tag{28}$$

for $f(\xi)$ we obtain the equation of the form

$$f_\xi^2 = \delta f^3 + \gamma_0 f^2 + \beta f + \alpha \tag{29}$$

with the coefficients

$$\begin{aligned} &\delta = -\frac{1}{3}\frac{\gamma_1}{\beta_1}, \qquad \gamma_0 = -\frac{\alpha_1}{\beta_1}, \qquad \alpha_1 = 6k_0 - l(l - 2\nu), \\ &\beta_1 = -(2l - 9k_0^2) + 3\nu l \qquad \gamma_1 = 6 + 6k_0 l[2\nu(\alpha_2 w_0)^{-1} + 1] - 3\nu l^2[\nu(\alpha_2 w_0)^{-1} + 1], \end{aligned} \tag{30}$$

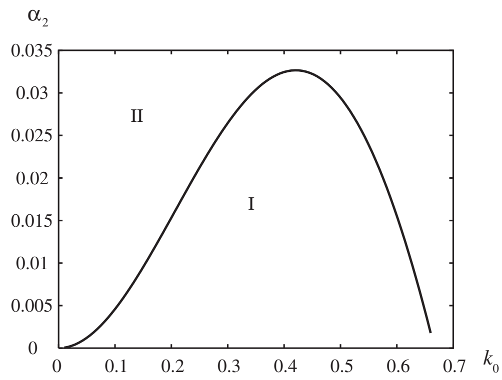

Figure 6. The range of values of the parameters α_2 and k_0 (I), which satisfy the long-wavelength approximation (33).

and where α, β are arbitrary constants. Let us analyse possible solutions of equation (29). For $\alpha = \beta = 0$ and with the inequality $\gamma_0 > 0$ fulfilled, equation (29) allows the formation of a solitary wave with the profile that has the form

$$f = -\frac{\gamma_0}{\delta}\frac{1}{\cosh^2\left(\frac{1}{2}\gamma_0^{1/2}\xi\right)}. \tag{31}$$

For the case of $\alpha_2 \to 0$ considered in [18] $\nu = 0, m^2 = 6k_0w_0, \delta = 2r(1+3k_0l), r^{-1} = 2l+2mn$ and $\gamma_0 = r(m^2 - n^2)$. For the solution of (31), the inequality (27) takes the form

$$\gamma_1^2(1 - \alpha_2 w_0 k_0) \gg 3\alpha_1 l. \tag{32}$$

In addition to (32), the condition for the long-wavelength approximation must be fulfilled (the typical size of the soliton $\Lambda^{-1} \approx \gamma_0^{-1/2}$ must be greater than the wavelength k_0^{-1}):

$$\left|\frac{\Lambda}{k_0}\right| \approx \left|\frac{\sqrt{\gamma_0}}{k_0}\right| \ll 1. \tag{33}$$

The inequalities (32) and (33) define the range of the parameters α_2 and k_0 values, in which the formation of the quasi-soliton (31) is possible.

The numerical analysis shows that, for the considered range of material parameter values, the inequality (32) is satisfied practically always. However, the approximation (33) is true not only for all values of α_2 and k_0, but only for the points with coordinates α_2 and k_0 lying lower on the curve in figure 6 (region I). The dependence of the quasi-soliton (31) width on α_2 and k_0 is presented in figure 7(a), and also the amplitude of the same parameters is shown in figure 7(b). When plotting the graphs in figures 6 and 7 we have assumed the parameter w_0 to be equal to unity.

Returning to the Goldstone modes (21), note that against the background of a plane wave (17), as is shown in [18], the most intensively interacting are the modes with $\varepsilon_1 = 1$. It is convenient to pass over to the frame of reference moving with the phase velocity of the waves $v_{01} = 3k_0^2 - 2\alpha_2 w_0 + m$ and to use new variables $\xi = X + v_{01}T$, $t = T$, instead of x and t, when the modulation of the wave (17) is determined by the excitation and interaction of only these modes. As long as the dependence of the field $\chi(\xi, T)$ on the variable T is weak in the case under consideration, we can neglect the terms $\partial_T^2\chi$ and $\partial_T\partial_\xi^3\chi$ in equation (26). As a result, equation (26) is reduced to the KdV equation:

$$\partial_T f + \frac{1}{2}\frac{\tilde{\beta}_1}{\tilde{\alpha}_1}\partial_\xi(f^2) + (1+n)\partial_\xi^3 f = 0 \tag{34}$$

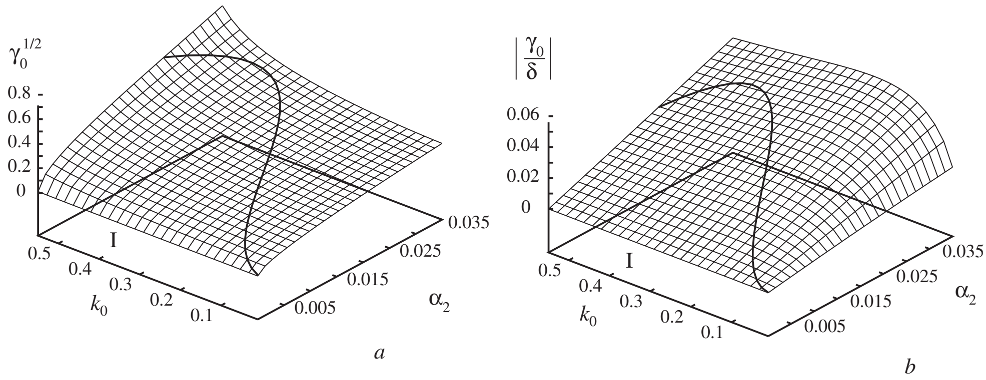

Figure 7. The dependence of the soliton width ($\gamma_0^{1/2}$) (30) on the parameters α_2 and k_0 (*a*); the dependence of the soliton amplitude ($|\frac{\gamma_0}{\delta}|$) (30) on the same parameters (*b*).

where $f = \partial_\xi \chi$, $\tilde{\alpha}_1 = -2\nu m(\alpha_2 w_0)^{-1}$ and $\tilde{\beta}_1 = 6 + 3\nu[m(\alpha_2 w_0)^{-1} - 1][4k_0 - \nu(m - \alpha_2 w_0)] + 2\nu[m(\alpha_2 w_0)^{-1} + 1][3k_0 - \nu(m - \alpha_2 w_0)]$. It can be easily verified that the solution (31) generalizes the one-soliton solution of the model (34). It follows from the analysis given above that low-amplitude modulation of spatially non-localized non-linear waves (with $\lambda < \Delta$) may lead to the formation of the specific 'dark' quasi-soliton with the amplitude (31) and the phase

$$\chi(\xi) = -\frac{2\gamma_0^{1/2}}{\delta} \tanh\left(\frac{1}{2}\gamma_0^{1/2}\xi\right). \tag{35}$$

5. Some exact solutions of ENSE and numerical modelling of multi-soliton states

It is known that each partial differential equation describes an unlimited variety of qualitatively similar phenomena or processes. This is evident from the fact that these equations have an infinite set of particular solutions. This is true also for equations of the type (9). For example, it is used in non-linear optics to describe the propagation of high-power femtosecond pulses of light, when the effects of the third order dispersion become comparable with the effects of the dispersion of the second order and its non-linear dispersion should be considered. In [34] the processes of Raman scattering are additionally taken into account, which are important in optics. Here in this section we discuss some analytical solutions of equation (9) and also we numerically investigate the problem of the evolution of the initial magnetization distribution specified in the form of pulses of various shapes.

As was noted above, equation (9) is not completely integrable at the arbitrary relation between the coefficients. The exceptions are the cases $\alpha_1 = \alpha_3 = 0$, when it is transformed into the classical NSE, and $\alpha_1 = 6\alpha_3$, when it is converted into a modified KdV equation (see, for instance, [25]). As for the special cases of the solutions, which describe the physical phenomena considered, they are chosen from the variety of particular solutions of the differential equation with the help of initial and boundary conditions. One of the primary attempts to develop a particular solution of equation (9), analogous to solutions of the classical NSE which correspond to the rapidly decreasing case ('bright' soliton) and the case of finite density ('dark' soliton), was undertaken in [35] where the special procedure was suggested which allowed us to write down explicitly the simple one-parameter soliton-like solutions and to define their domain of existence.

These solutions of ENSE (9) were called ‘dark’ and ‘light’ solitons by analogy with the localized states of NSE. A year later, analytical solutions similar to the solutions of this type were obtained by Potasek and Tabor [36]. They can be written in the following forms:

$$\Phi(X,t) = a\cosh^{-1}(b(X-Vt))\exp[\mathrm{i}(\kappa X-\omega t)] \tag{36}$$

with the parameters

$$\begin{aligned} &V = 2p\kappa - 3\alpha_3\kappa^2 + \alpha_3 b^2, \qquad \omega = p\kappa^2 - \alpha_3\kappa^3 - (p-3\alpha_3\kappa)b^2 \\ &\kappa = \frac{2p\alpha_1 - 2\alpha_3}{4\alpha_3\alpha_1}, \qquad \frac{a^2}{b^2} = 2\frac{\alpha_3}{\alpha_1} \end{aligned} \tag{37}$$

and

$$\Phi(X,t) = a\tanh(b(X-Vt))\exp[\mathrm{i}(\kappa X-\omega t)] \tag{38}$$

with the parameters

$$\begin{aligned} &V = 2p\kappa - 3\alpha_3\kappa^2 + 4\alpha_3 b^2 + 3\alpha_1 a^2, \qquad \omega = p\kappa^2 - \alpha_3\kappa^3 - (1-\alpha_1\kappa)a^2 \\ &\kappa = \frac{2p\alpha_1 - 2\alpha_3}{4\alpha_3\alpha_1}, \qquad \frac{a^2}{b^2} = -2\frac{\alpha_3}{\alpha_1} \end{aligned} \tag{39}$$

where $p = \frac{1}{2}\,\mathrm{sgn}(D_\omega)$. Retaining the terminology adopted in [11], we call these analytical solutions (36) and (38) the Potasek and Tabor soliton solutions (PT solitons); (36) is called the ‘bright’ PT soliton and (38) a ‘dark’ PT soliton. These solutions represent solitary waves which do not change their profile during propagation and have a finite energy. However, they can hardly be treated as real solitons in the sense of inverse scattering problem terms [35]. Note that the solution corresponding to the case of finite density is not built up in [34] (see also [14]).

Note that the existence of both solutions (36) and (38) of equation (9) is possible under the condition $a^2/b^2 > 0$. This quantity is determined by the ratio of the third-order dispersion and the dispersion of non-linearity ($2\alpha_3/\alpha_1$). The range of negative values of $2\alpha_3/\alpha_1$ corresponds to ‘dark’ solitons and that of the positive values to ‘bright’ ones. The calculated wavenumber kd dependence of the $2\alpha_3/\alpha_1$ for surface magnetostatic waves for various values of the ratio Δ/d is presented in figure 8. For all curves in this picture with various values of Δ/d the range of small kd values corresponds to ‘bright’ PT solitons. With increasing kd the range appears in which $2\alpha_3/\alpha_1 < 0$ and ‘dark’ PT solitons are realized. This situation differs principally from the case of volume magnetostatic waves in a tangentially magnetized film where the small kd range corresponds to ‘dark’ PT solitons and the range of larger values of kd to the ‘bright’ ones [12]. With increasing relative interlayer thickness Δ/d the stability domain of bright solitons narrows down. For instance, if $\Delta/d = 1$ the ratio $2\alpha_3/\alpha_1$ is positive for $kd < 0.63$ and for $\Delta/d = 10$ it is positive only for the values of $kd < 0.15$.

Below, using numerical methods in the framework of equation (9) we shall study the evolution problem of the initial magnetization distribution which is given in the form of the impulses of different shapes. These equations will be solved in the domain of existence of ‘bright’ solitons at zero boundary conditions at infinity and the initial distribution chosen in the form of a localized pulse with a soliton-like envelope:

$$\Phi(X,0) = \frac{A}{\cosh(B(X-X_0))} \tag{40}$$

and with an envelope close to a Gaussian distribution:

$$\Phi(X,0) = A\exp\left[-\frac{B^2}{2}(X-X_0)^2\right] \tag{41}$$

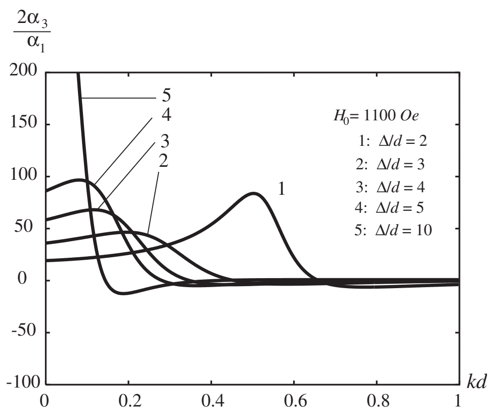

Figure 8. The wavenumber dependence of the coefficient $2\alpha_3/\alpha_1$ for surface magnetostatic waves at several values of Δ/d, $M_0 = 1740$ G, $d = 14$ μm.

with various characteristic parameters A and B, where A is the amplitude and B^{-1} is an effective width of the distribution. The choice of initial distribution in the form given by (40) is trivial. Such a distribution coincides in essence with the form of a single soliton. If we take the values of the parameters A and B in this distribution close to the exact relations (37) then the formation of a 'bright' PT soliton occurs as follows. At the beginning the radiation is split off from the starting pulse. This radiation rapidly spreads out and one 'pure' soliton is formed in the system. All parameters of this soliton, naturally, satisfy the relations (37). The aforesaid refers to both the distributions (40) and (41). The shape of the distribution that is obtained as a result of the evolution of (40) or (41) substantially depends on the parameter ξ, which is defined as follows: $A^2/B^2 = \xi 2\alpha_3/\alpha_1$. Recall that the parameter α_3 describes the third-order dispersion and the parameter α_1 describes the non-linearity dispersion. In our opinion, the parameter ξ here plays the same part as the parameter $(n - 1/2)^2$, where n is the number of generated solitons, which was introduced in [21] (see also [2], relation (14)). The situation that was investigated in [2] and [21] was analytically described by the classical NSE. The parameter $(n - 1/2)^2$ was the coefficient that related the product of the distribution amplitude by the distribution width with the ratio of the second-order dispersion to the non-linearity. In this context ξ relates the ratio of the distribution amplitude to the inverse width of the distribution with the ratio of third-order dispersion to the non-linearity dispersion. Note also that, for $\xi = 1$, the distribution of the form (40) is the exact solution. We have investigated the situations in which the parameter ξ runs within the relatively wide range of values from very small ($\ll 1$) to about ten.

If we take the distribution with a small amplitude at given α_3 and α_1 such that $\xi \ll 1$, then, as a rule, a single soliton is formed in the system (see figure 9) with characteristics that are defined by the relations (37) both for the soliton-like (40) and for the Gaussian (41) initial pulses. In such an approach we did not find the threshold of the single-soliton formation up to extremely small values of ξ. We succeeded in establishing that, for the soliton-like initial distribution, the single 'bright' soliton is formed for ξ belonging to the interval $0 < \xi < 3.4$, and for the Gaussian distributions it is formed for ξ taken from a wider interval $0 < \xi < 6$.

In the case of large ξ values the following scenario is realized. If one takes the amplitude value of the initial distribution considerably exceeding the amplitude of the exact solution ($\xi > 1$), not one but several solitons may be generated in the system and the multi-soliton state is formed in a threshold way. For given values of α_3 and α_1 the intervals of the ξ values can

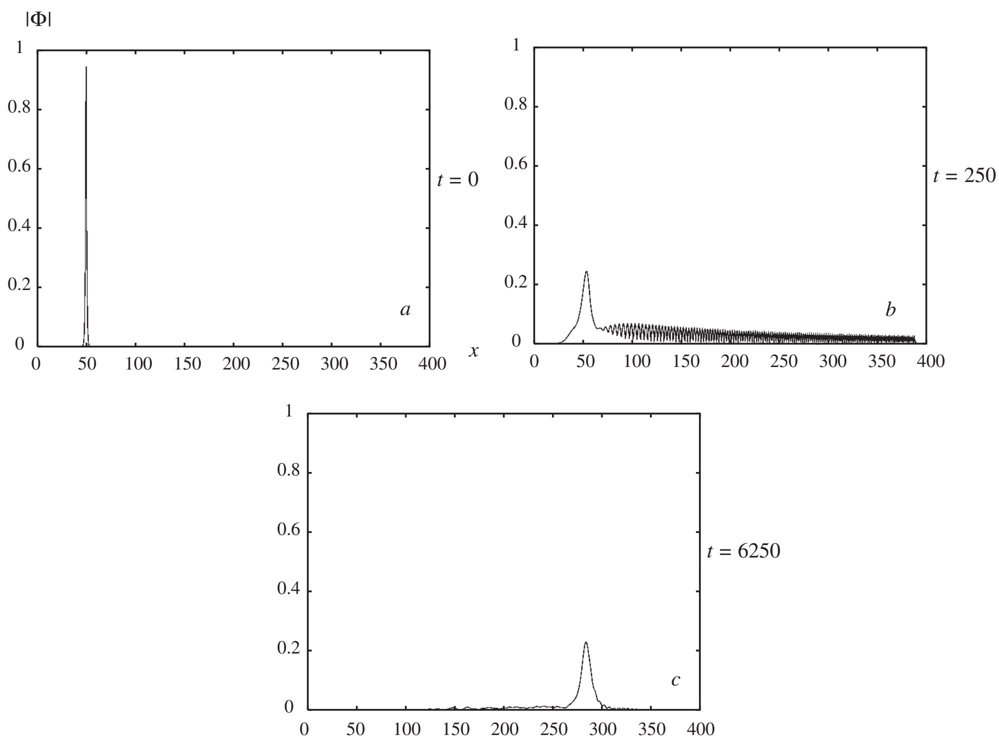

Figure 9. The evolution of the initial distribution (40) to the one-soliton state for $\xi = 0.1$. The parameters of the system are $\Delta/d = 5$, $kd = 0.27$, starting distribution (*a*), initial stage of distribution evolution (*b*) and the just formed soliton (*c*).

be determined in which, say, two, three or more solitons necessarily originate (see figure 10). Unfortunately, we were unable to determine the upper limit of the range of formation of three solitons. Figure 10 illustrates the process of formation of two solitons from the Gaussian distribution. Two solitons are formed from the distribution (40) for ξ, belonging to the interval $3.4 < \xi < 10$, and from the Gaussian distribution (41) for ξ lying in the interval $6 < \xi < 20$. It is evident from figure 10 that the velocities of propagation of the just-formed solitons may differ not only in value, but also in sign as well. It should be emphasized that we keep in mind that the velocity is measured in the frame of reference moving with the group velocity.

One of the first attempts to solve the Cauchy problem for equation (9) was undertaken in [17]. Equation (9) has been solved at the zero boundary conditions and for the initial distribution in the form of the localized envelope similar to (40). In this calculation all coefficients of equation (9) were put equal to unity. It was shown that, if the amplitude and width of the initial distribution exceeds some threshold values, several soliton-like states are excited in the system. These threshold values of the amplitude and width of the initial distribution corresponding to two, three and four soliton states were obtained. The question of the character and properties of the distributions obtained remains open. The numerical calculations carried out by us differ in two points. First, the coefficients of equation (9) were taken as the values corresponding to the real high dispersion system, that is, the three-layer film. The second, and principal point is we happen to identify the states obtained as the Potasek–Tabor solitons. The characteristic parameters of these solitons are described by the relations (37). Such an identification was not done in [17].

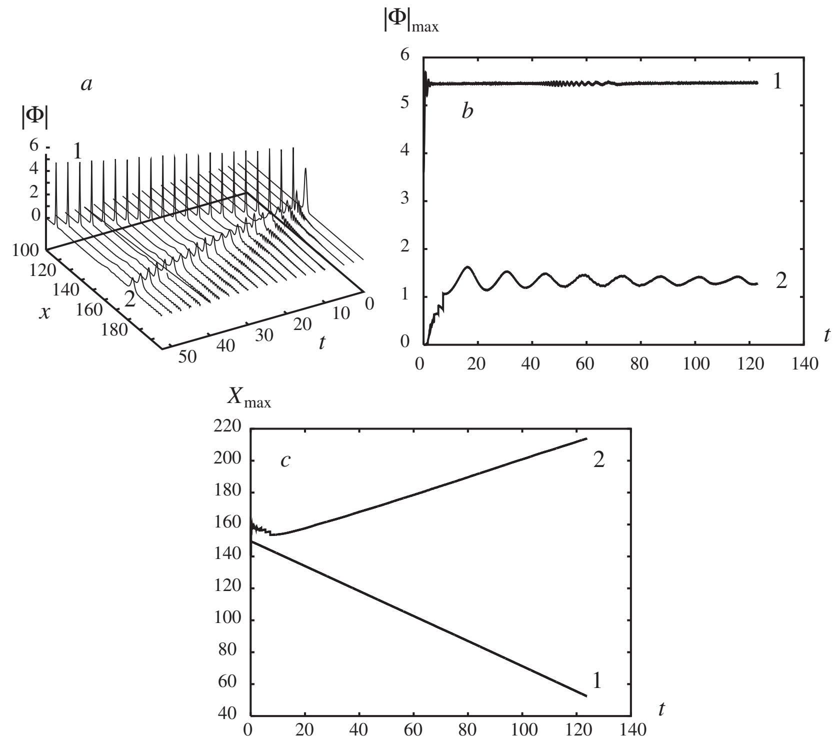

Figure 10. The formation of a two-soliton state from the initial distribution (41) for $\xi = 7.0$ (*a*), the time dependence of the amplitude of the first and second solitons (*b*) and the time dependence of the coordinates of the solitons (*c*).

6. Conclusion

In the framework of the ENSE, taking account of both the third-order dispersion and the non-linearity dispersion, the peculiarities of the generation, interaction and evolution of envelope solitons of surface magnetostatic waves in a layered ferromagnet–dielectric–metal structure are studied. It is shown that the dispersion curves of these waves demonstrate non-monotonic behaviour that is transformed with the variation of dielectric interlayer thickness Δ. For all of this, the value $\Delta = k^{-1}$ may be the 'zero-dispersion' point of the spectrum in a definite range of layer thicknesses and dc magnetic field. The transformation of the dispersion curve allows us to control effectively the peculiarities of the non-linear regime of the wave's propagation, changing the 'bright' soliton-like regime to the 'dark' soliton-like and vice versa. It is shown also that, in the range of small kd values, the 'bright', but not 'dark', Potasek–Tabor solitons are realized in the case of the surface waves considered, contrary to the volume waves (and the same in the tangentially magnetized structure).

The process of low-amplitude modulation of spatially non-localized non-linear waves is studied in the vicinity of the 'zero-dispersion' point $\Delta = k^{-1}$. It is shown that the modulation of the waves with the wavelength $\lambda < \Delta$ can lead to the formation of specific 'dark' quasi-solitons described by the KdV equation. In contrast to this, the waves with the wavelength $\lambda > \Delta$ are modulation-unstable with their amplitude exponentially increasing in the vicinity of the 'zero-dispersion' point. The numerical analysis of the conditions of formation of the multi-soliton states shows that a certain amplitude of the initial distribution should be reached for the excitation, while no such threshold exists for single-soliton states.

We hope that the present analytical and numerical findings will help both in the experimental search for new stable localized states of the envelope surface waves and in the investigation of the peculiarities of their origination, interaction and transformation occurring with changes in the material parameters of the layered metallized structure. In this respect, the ferromagnet–dielectric–metal structure is a unique model system with variable dispersion and non-linear properties.

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